(L-2)

Order axioms in $\mathbb{R}$

In the set of real number $\mathbb{R}$ the following axioms are satisfied known as order Axioms.

(1) Law of trichotomy:-
for $a, b \in R$ exactly one of the followings holds:
(i) $a > b$ (ii) $a = b$ (iii) $b > a$.

(2) transitivity:- for $a, b, c \in \mathbb{R}$, $a > b$ and $b > c$ $\Rightarrow a > c$.

(3) monotone law of addition:-
for $a, b \in \mathbb{R}$: $a > b \Rightarrow a + c > b + c$: $\forall c \in R$.

(4) monotone law of multiplication:-
for $a, b \in \mathbb{R}$: $a > b$ and $c > 0 \Rightarrow ac > bc$
& $d < 0 \Rightarrow ad < bd$.

In view of the above axioms $(\mathbb{R}, +, \cdot)$ is called ordered field. Similarly $(\mathbb{Q}, +, \cdot)$ is also an ordered field.

Interval: let a & b be two ~~element~~ real no.s with $a < b$. then

(1) the set $\{x : x \in \mathbb{R}, a \le x \le b\} = [a, b]$ is called a closed intervals. a and b are called the end points of the interval. a is called the left end point while b is called the right end point. Thus in a closed interval, both end points belong to the interval.
i.e. $[2, 3]$, $[4, 7]$.

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(ii) The set $\{x : x \in R : a < x < b\} = (a, b)$ is called an open intervals & is denoted by (a, b). In an ~~order~~ open intervals, the end points of the intervals, a is called the left end point while b is called the right end point. Thus a and b do not belong to the interval

Remark:- The intervals which is open from one side and closed from another side is called semi-open or semi-closed interval
e.g. :- $(3, 5]$ or $[5, 9)$.

* Length of an interval:- For each intervals with end points a & b $(a<b)$, $b-a$ is called the length of the intervals. $[a, b]$, (a, b), $[a, b)$ $[a, b]$ has the same length $b-a$!

* Finite and Infinite Intervals:- An intervals is said to be finite or infinite according as its length is finite or infinite. Thus the intervals $[2, 3)$, $(2, 3)$, $[2, 3)$, $(2, 3]$ are finite because its length $3-2 = 1$ is finite. The interval $[1, \infty)$, $(1, \infty)$, $(-\infty, 1)$ are infinite intervals.

(not closed)

Note:- Every intervals except ϕ and singleton is an infinite set but every infinite set need not be an intervals.
eg :- $\mathbb{N}$ is an infinite set but not interval.

(2) Ordered pair:- ordered pair is an elements of the form (a, b). The element a is called first element and the element b is called 2nd element of ordered pair

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* Equality of ordered pairs :

Let (a,b), (c,d) be any two ordered pairs then $(a,b) = (c,d) \Leftrightarrow a = c ; b = d$.

* Cardinally equivalent or equivalent sets :-

A set 'A' is said to be equivalent or cardinally equivalent to the set B. If $\exists$ one-one mapping from set A to set B. This relation is denoted by the symbol '$\sim$'

eg:- $A = \{1,2,3\}$, $B = \{4,5,7\}$

$A \sim B$.

* Cardinal numbers :-

The cardinal number of an equivalence sets is any representative of the class or we can say that every equivalence class defines a unique cardinal number. If S is any set consisting of 5-elts then the cardinal number of S is 5. The cardinal number of empty set ϕ is defined as zero. If $A \sim \{1,2,3,--,n\}$, then n is called the cardinal number of set A. and is denoted by Card A or $|A|$, thereafter

$B = \{1,3\}$, $|B| = 2$, $|\phi| = 0$.

$\rightarrow$ (a,b), $[a,c]$, $[b,d)$, $(e,f]$ are equivalent

* Sum of Cardinal number :-

Let A and B be any two set with cardinality n and m such that their intersection is empty.

Let $|A| = n$, $|B| = m$ and $\{A \cap B\} = \phi$. thus

thus $|A \cup B| = n + m$.

eg:- If $A = \{1,2,3\}$, $B = \{3,4\}$. $A \cap B \neq \phi$

thus $A \cup B = \{1,2,3,4\}$, $|A| = 3$, $|B| = 2$

$|A \cup B| = 4$.

By the definition $|A \cup B| \neq |A| + |B|$

or $|A \cup B| \leq |A| + |B|$.

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* Cartesian Product :- Let A and B be two sets. Then $A \times B = \{(x,y) : x \in A, y \in B\}$ is known as Cartesian product of A and B. In general $A \times B \neq B \times A$ where

$B \times A = \{(y,x) : y \in B, x \in A\}$

Let $A = \{1,2\}$, $B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$B \times A = \{(3,1), (4,1), (3,2), (4,2)\}$, $A \times B \neq B \times A$

Properties of Cartesian product

(i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

(iii) $(A - B) \times C = (A \times C) - (B \times C)$

imp (iv) $\boxed{A \times B = \phi \Leftrightarrow A = \phi \text{ or } B = \phi}$

imp (v) $\boxed{A \times C \subseteq B \times C, C \neq \phi} \Rightarrow A \subseteq B$ (Right Cancellation)

imp (vi) $A \times B = B \times A \Leftrightarrow A = \phi$ or $B = \phi$ or $A = B$.

(vii) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

(viii) $A = A \times B \Rightarrow A = \phi$

(ix) $A \subseteq A \times A \Rightarrow A = \phi$.

(x) If $\left.\begin{array}{l} A \text{ contains } n \text{ elts} \\ B \text{ contains } m \text{ elts} \end{array}\right\} \Rightarrow A \times B$ contains nm elts.

* Relations :- A relation from set A to set B is a subset of $A \times B$.

Ily a relation from set B to set A is a subset of $B \times A$

eg :- $A = \{1,2,3\}$, $B = \{2,3\}$ then

$A \times B = \{(1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\}$

is relation from A to B.

Ily $B \times A = \{(2,1), (3,1), (2,2), (3,2), (2,3), (3,3)\}$

is a relation from B to A.

Thus every subset of $A \times B$ ~~and $B \times A$~~ is a relation from A to B.

Ily every subset of $B \times A$ is a rel^n from B to A.

since ϕ is a subset of every set. Hence ϕ is a relation from A to B (and B to A) known as empty relation.

Remark:- If a set A has n element and set B has m elements, then total number of relations from set A to set B is
$$2^{n \times m} = 2^{n \cdot m} = 2^{|A| \cdot |B|}.$$

types of Relation

(1) Inverse Relation:- let R be a relation on A. Let $R = \{(1,2),(4,1)\}$ then

inverse $R^{-1} = \{(2,1),(1,4)\}$

(2) Identity Relation:- A relation R in a set A is an identity relation if $I_A = \{(x,x) : x \in A\}$

eg:- $A = \{1,2\}$, $R = \{(1,1),(2,2)\}$

(3) Universal Relation:- If R is a relation in a set A, then R is a universal relation if $R = A \times A$.

(4) Void relation:- If R relation in set A. then R is called void relation if $R = \phi$

Properties of relation

Let R be a relation on A then

(a) R is called reflexive:- if $(a,a) \in R$ $\forall a \in A$

eg:- $A = \{1,2,3\}$, $R = \{(1,1),(2,2),(3,3)\}$.

(b) R is symmetric:- If $(a_1,a_2) \in R \Rightarrow (a_2,a_1) \in R$